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TUNNEL-WALL CORRECTIONS TO ROLLING AND YAWING MOMENTS DUE TO AILERON DEFLECTION IN CLOSED RECTANGULAR WIND TUNNELS

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TUNNEL-WALL CORRECTIONS TO ROLLING AND YAWING MOMENTS DUE TO AILERON DEFLECTION IN CLOSED RECTANGULAR WIND TUNNELS

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SUMMARY

A method is developed for calculating the tunnel-wall corrections to rolling and yawing moments due to aileron deflection on models in closed rectangular wind tunnels. Graphs are presented which permit a rapid determination of these corrections for models mounted in 7- by 10-foot or 8- by 12-foot wind tunnels. The method is so developed that the corrections may be calculated for the deflection of either aileron alone or both ailerons simultaneously.

INTRODUCTION

In the prediction of the aerodynamic characteristics of airplanes in free flight from wind-tunnel tests of scale models, it is necessary first to evaluate the modifying effect of the tunnel walls on the forces and moments experienced by the models. This general problem of wind-tunnel-wall interference effects accordingly has become the subject of many investigations.

It often is desired to predict the rolling and yawing characteristics of airplanes in free flight from wind-tunnel tests. The effects of the presence of the tunnel walls on the rolling and yawing moments measured in the wind tunnel are therefore of particular interest. Wind-tunnel-wall effects on the rolling and yawing moments due to aileron deflection have been analyzed and corresponding corrections calculated by Koning and Van der Maas (reference 1) for a model with ailerons extending to the wing tips in a circular wind tunnel. Stewart (reference 2) has extended the analysis of the tunnel-wall interference on the yawing moment due to aileron deflection to the case of ailerons set in from the wing tips for circular wind tunnels. Swanson (reference 3) has investigated the effects of the tunnel walls on the major forces and moments acting upon yawed models in closed rectangular wind tunnels. In the present paper an analysis is made of the modifying effects of the presence of tunnel walls on the rolling and yawing moments due to aileron deflection for models in closed rectangular wind tunnels. Corrections are developed by which the rolling and yawing moments determined from the wind-tunnel tests may be adjusted to equivalent free-air values.

It should be noted that it is possible to obtain similar corrections from the results of the analysis of reference 3. This analysis, however, was undertaken principally to investigate the validity of the assumption commonly made in practice that the tunnel-wall corrections which are applied to unyawed models may also be applied without modification to yawed models. It consists therefore of a comparison of the tunnel-wall corrections calculated for yawed models with the corresponding corrections for unyawed models and is not advanced as a general method for computing these corrections for unyawed models. The computational procedure involved in the evaluation of these corrections for any unyawed model is considered too laborious to permit the general application of the method of reference 3 in practice.

The following development has been so simplified by the inclusion of convenient nondimensional parameters and graphs as to permit a very rapid determination of the tunnel-wall corrections to the rolling and yawing moments due to aileron deflection for any model in closed rectangular wind tunnels having height-to-breadth ratios of 0.700 or 0.667. These height-to-breadth ratios (for 7- by 10-ft and 8- by 12-ft wind tunnels) are by far the most commonly found in practice. The corrections derived by the method of this report are not appreciably affected by small differences in test section proportions; hence the graphs presented may be applied with sufficient accuracy to wind tunnels with height-to-breadth ratios in the neighborhood of 0.700 and 0.667, respectively. For wind tunnels of relative proportions markedly different from 7 by 10 feet or 8 by 12 feet, the general method may be applied to obtain the corrections for a particular model.

SYMBOLS

Γ vortex strength, magnitude of circulation

Γ_w wing circulation

Γ_a aileron circulation

V free-stream velocity

w induced upwash velocity

w' upwash velocity induced at center of wing by images of wing lift distribution

w'' upwash velocity induced at any spanwise station of wing by images of aileron lift system

S wing area

s wing semispan

s_a aileron span

c chord

$\bar{c}$ mean aerodynamic chord

a_1 distance from plane of symmetry to inboard end of aileron

a_2 distance from plane of symmetry to outboard end of aileron

A aspect ratio

λ taper ratio

b wind-tunnel breadth

h wind-tunnel height

y spanwise distance from plane of symmetry

C_L lift coefficient

c_l section lift coefficient

C_l' measured rolling-moment coefficient

ΔC_l correction to rolling-moment coefficient

C_l corrected rolling-moment coefficient

C_n' measured yawing-moment coefficient due to aileron deflection

ΔC_n correction to yawing-moment coefficient due to aileron deflection

C_n corrected yawing-moment coefficient due to aileron deflection

m_o section lift-curve slope (per radian)

ρ mass density of air

THEORY

The problem presenting itself for analysis is separable into two parts: first, the analysis of the effect of the tunnel walls on the flow over the model with its corresponding effect on the measured characteristics; and second, the evaluation of corrections to modify these characteristics to their equivalent free-air values.

Consider a wing of finite span mounted in a closed-throat wind tunnel. If the tunnel walls be replaced by a suitable system of images of the wing as in figure 1, it can be seen that the walls induce velocities over the wing which are not present in an unconfined air stream. These velocities are of directions which alter the flow of air past the wing, thereby affecting its aerodynamic characteristics. The effects of these tunnel-wall induced velocities on measured characteristics have been analyzed by many investigators and are well known. If an aileron is now deflected on the wing, the effect of the tunnel walls is to induce a velocity over the wing which is proportional to the circulation about the aileron. This velocity is variable across the span and therefore distorts the flow pattern about the wing in an unsymmetrical manner, producing rolling and yawing moments which do not exist in free air. Another induced yawing moment results from the interaction of the resultant tunnel-wall induced velocity existing before aileron deflection and the circulation about the deflected aileron.

The steps in the evaluation of the induced rolling and yawing moments are laid down as follows: The tunnel-wall induced upwash velocity acting over the wing is first determined. The upwash velocity induced by the walls due to a downward deflection of the right aileron alone is next determined. From these two induced velocities expressions for the induced rolling and yawing moments are developed. The final numerical evaluation of the induced moments is dependent upon the respective dimensions of the model and the wind tunnel in which the test is made.

In any determination of induced velocities and consequent induced aerodynamic forces and moments, the type of lift distribution over the wing is of prime consideration. This distribution may be different for every model tested, but can be closely approximated for most models by an elliptical type of distribution. A rigorous mathematical development of expressions for the induced yawing moments for an elliptical distribution of lift, however, is precluded by the complexity and general inapplicability in practice of the results obtained by such a development. It is possible by making several simplifying assumptions in the mathematical treatment to reduce the results to a practicable form without affecting the basic analysis in any way. Accordingly, in the following development the simplifying assumptions are made of a uniform lift distribution and a constant spanwise distribution of the tunnel-wall induced velocity existing before aileron deflection. Although these assumptions are strictly valid only for models of span less than half the tunnel breadth, they permit a very satisfactory determination to be made of the manner in which the induced yawing moments vary with model and wind-tunnel dimensions.

Induced yawing moments corresponding to the elliptical type of lift distribution are subsequently evaluated numerically for several model and tunnel configurations selected so as to cover the complete range of practical installations. Models with span varying within the approximate limits of 70 to 90 percent of the tunnel breadth are investigated in this manner, and the induced moments so calculated plotted as functions of the model and tunnel dimensions. From these calculations corrective coefficients have been determined which modify the first approximation of the induced moments to a degree closely approximating the values corresponding to actual loading conditions. These corrective coefficients are applied to the final expressions developed for the induced yawing moments.

INDUCED UPWASH VELOCITIES

Consider a wing mounted in a closed rectangular wind tunnel so that the model axes are coincident with the wind axes. A system of coordinates (x, y, z) of the model is chosen so that the axis of x is parallel to the direction of air flow, the axis of y is horizontally perpendicular to the direction of flow and located in the plane of the wing quarter chord, and the axis of z is located vertically in the plane of the wing quarter chord (fig. 2). The right aileron alone is deflected downward.

Let it be assumed that the wing can be replaced by a lifting horseshoe vortex of strength Γ_w. Similarly, let it be assumed that the unsymmetrical vortex system produced by the deflected aileron can be treated as an elementary airfoil with constant circulation Γ_a and a pair of trailing vortices of strength Γ_a. The trailing-vortex system then consists of the principal trailing vortices existing before the aileron deflection and the eccentrically located aileron trailing vortices. To satisfy, in the analysis, the boundary requirement of zero normal velocity at the walls, a doubly infinite series of images of the vortex system is introduced (fig. 1). For simplicity in further treatment, the aileron system alone is shown in figure 3.

The upwash velocity induced at the lifting line by the images of the uniform spanwise lift distribution has been calculated for closed rectangular wind tunnels by Glauert (reference 4) and Rosenhead (reference 5) as

$$w' = C_L \frac{SV}{8b^2} \left[\frac{\pi}{3} \; 8\pi \sum_{p=1}^{\infty} \frac{p}{1 + e^{2\pi p h/b}} \right] \tag{1}$$

The normal velocity induced at the center of the lifting line by an image of the aileron trailing vortex of strength Γ_a located at the point (y, z) is

$$w''_{y_1} = \frac{\Gamma_a}{4\pi}\left(\frac{y}{y^2 + x^2}\right)$$

Similarly, the upward component of velocity induced at any point y of the lifting line by all of the images of the aileron vortex system for a downward aileron deflection may be written as

$$w'' = \frac{\Gamma_a}{4\pi}\left\{\sum_{n=1}^{\infty} (-1)^n \left[\frac{1}{nb + (-1)^n a_1 - y} - \frac{1}{nb + (-1)^n a_2 - y}\right.\right.$$

$$\left. + \frac{1}{nb - (-1)^n a_2 + y} - \frac{1}{nb - (-1)^n a_1 + y}\right]$$

$$+ 2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} (-1)^{m-1} (-1)^n \left[\frac{nb + (-1)^n a_2 - y}{(mh)^2 + (nb + (-1)^n a_2 - y)^2}\right.$$

$$- \frac{nb + (-1)^n a_1 - y}{(mh)^2 + (nb + (-1)^n a_1 - y)^2} + \frac{nb - (-1)^n a_1 + y}{(mh)^2 + (nb - (-1)^n a_1 + y)^2}$$

$$\left. - \frac{nb - (-1)^n a_2 + y}{(mh)^2 + (nb - (-1)^n a_2 + y)^2}\right]$$

$$\left. + 2\sum_{m=1}^{\infty} (-1)^{m-1}\left[\frac{a_2 - y}{(mh)^2 + (a_2 - y)^2} - \frac{a_1 - y}{(mh)^2 + (a_1 - y)^2}\right]\right\} \quad (2)$$

By substituting for one infinite series its sum of the general form

$$\sum_{m=1}^{\infty} (-1)^{m-1} \frac{x}{m^2h^2 + x^2} = \frac{1}{2x} - \frac{\pi}{2h} \operatorname{csch} \frac{\pi}{h} x$$

equation (2) may be simplified to

$$w'' = \frac{\Gamma_a}{4\pi}\left\{\frac{1}{a_2 - y} - \frac{1}{a_1 - y} + \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a_1 - y) - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a_2 - y)\right.$$

$$+ \sum_{n=1}^{\infty}(-1)^n\left[\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb + (-1)^n a_1 - y\right)\right.$$

$$- \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb + (-1)^n a_2 - y\right) + \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb - (-1)^n a_2 + y\right)$$

$$\left.\left. - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb - (-1)^n a_1 + y\right)\right]\right\} \tag{3}$$

ROLLING-MOMENT CORRECTION

By virtue of the Kutta-Joukowski vortex theory of lift, the rolling moment produced by the positive circulation of the deflected right aileron may be expressed as

$$L = \rho V\int_{a_1}^{a_2}\Gamma_a\, y\, dy \tag{4}$$

Since the aileron circulation Γ_a has been assumed to be constant over the aileron span, equation (4) becomes, after integration

$$L = \frac{\rho V\Gamma_a}{2}\left(a_2{}^2 - a_1{}^2\right)$$

which in coefficient form is

$$C_l = \frac{\Gamma_a}{2VSb}\left(a_2{}^2 - a_1{}^2\right) \tag{5}$$

The induced upwash velocity w'' effectively increased the angle of attack of the wing by the angle w''/V, where V is the velocity of the undisturbed stream. The increase in lift of any section brought about by this change may be expressed as

$$\Delta c_l = m_o \frac{w''}{V}$$

where m_o is the slope of the section lift curve (per radian). Because the induced velocity w'' is variable across the span, the wing may be considered to be effectively twisted by the angle w''/V, and subject to the consequent rolling-moment increment

$$\Delta L = \frac{A}{A + 4} m_o \frac{\rho V}{2} \int_{-s}^{s} w'' cy\, dy \qquad (6)$$

The factor $\frac{A}{A + 4}$ is introduced in this expression because the rolling moment actually experienced by a wing twisted through the induced upwash angle w''/V is approximately $\frac{A}{A + 4}$ times the rolling moment calculated for a wing of infinite aspect ratio (reference 6, p. 103). This factor holds strictly for wings of elliptical plan form only, but is considered sufficiently accurate for wings of other plan form in view of the fact that the rolling-moment correction is but a small fraction of the total moment.

A numerical evaluation of the induced rolling moment from equation (6) for the general case of a wing with variable chord is not feasible because the spanwise variation of wing chord is different for almost every model tested. To avoid this complication the mean aerodynamic chord is substituted for c in equation (6). This assumption of constant chord permits the integration of the right-hand side of the equation for the general case without further difficulty. It now remains to account for the modifying effect of a variable chord on the value of the rolling-moment increment calculated for a constant mean aerodynamic chord.

Equation (6), or its equivalent, has been numerically evaluated for several model and wind-tunnel configurations comprising the complete range of practical installations - that is, for model spans ranging from approximately 70 to 90 percent of the tunnel breadth, and for taper ratios from zero to unity. The results of these calculations have been compared with the corresponding induced rolling moments as determined by the method of this report for constant mean aerodynamic chords. From these comparisons the correction factor K_1 has been determined as a function of taper ratio (fig. 4), and is introduced in equation (6) to adjust the induced rolling moment calculated from the graphs of this report for the mean aerodynamic chord to very nearly the actual value experienced by a wing of any particular plan form. The variation of K_1 with the ratio of model span to tunnel breadth was found to be negligible over the range of ratios investigated. After making the indicated substitutions, equation (6) becomes

$$\Delta L = K_1 \frac{A}{A+4} m_0 \frac{\bar{c}\rho V}{2} \int_{-s}^{s} w'' y dy$$

The indicated integration is complicated and tedious and is therefore performed in the appendix. The result has been numerically evaluated for all practical models having the relative proportions of 7- by 10-foot and 8- by 12-foot wind tunnels and has been presented graphically in figures 5 and 6, respectively. As a convenient simplification, the nondimensional parameter $F_1\left(\frac{a}{b}\right)$ is introduced. Reducing the rolling-moment increment to coefficient form, equation (7) therefore becomes

$$\Delta C_l = K_1 \frac{A}{A+4} \frac{m_0}{8\pi} \frac{\bar{c}b\Gamma_a}{V s S} \left[F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right)\right] \tag{8}$$

where (cf. appendix)

$$F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right) = \frac{4\pi}{b\Gamma_a} \int_{-s}^{s} w'' y dy$$

The expression for the induced rolling-moment coefficient may be rewritten as

$$\frac{\Delta C_l}{C_l} = K_1 \frac{A}{A+4} \frac{m_0 \bar{c}b}{4\pi(a_2^2 - a_1^2)} \left[F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right)\right] \tag{9}$$

In the evaluation of the rolling-moment correction (equation (9)), for a particular model, the coefficient K_1 is determined from figure 4 and values of $F_1\left(\frac{a}{b}\right)$ are taken from figures 5 and 6.

The relation between the free-air rolling-moment coefficient and the wind-tunnel rolling-moment coefficient is then finally

$$C_l = \frac{C_l'}{1 + \frac{\Delta C_l}{C_l}} \tag{10}$$

YAWING-MOMENT CORRECTION

Again in accordance with the Kutta-Joukowski vortex theory, a general expression for the induced yawing moment about a plane of symmetry produced by a circulation between the limits of y and $-y$ may be evolved as

$$N_i = \rho \int_{-y}^{y} w\Gamma y\,dy$$

or in coefficient form

$$C_{n_i} = \frac{1}{SV^2 s} \int_{-y}^{y} w\Gamma y\,dy$$

In the following development the induced yawing moment due to the deflection of the right aileron alone on a wing in a closed rectangular wind tunnel is split into three increments each of which is treated separately. The upwash velocity w' induced by the images of the uniform lift distribution, and the aileron circulation Γ_a produce a lower drag over the aileron span than that existing before aileron deflection. This asymmetrical reduction of drag produces a yawing moment the coefficient of which is denoted by ΔC_{n_1}. The upwash velocity w'' induced by the images of the aileron lift system and the constant circulation Γ_w about the wing produce a yawing-moment increment which in coefficient form is denoted by ΔC_{n_2}. A third yawing-moment increment correspondingly denoted by ΔC_{n_3} is produced by the induced upwash velocity w'' and the aileron circulation Γ_a.

Yawing-moment correction ΔC_{n_1}.-

$$\Delta C_{n_1} = \frac{1}{SV^2 s}\int_{a_1}^{a_2} w'\Gamma_a \, y dy \qquad (11)$$

Since Γ_a has been assumed constant, substitution of equation (1) for w' in equation (11) yields

$$\Delta C_{n_1} = C_L \frac{\Gamma_a}{8b^2 Vs}\left[\frac{\pi}{3} + 8\pi \sum_{p=1}^{\infty} \frac{p}{1 + e^{2\pi ph/b}}\right]\int_{a_1}^{a_2} y dy \qquad (12)$$

As discussed previously in this report the actual lift distribution is not uniform and the induced upwash velocity is not constant across the span. Accordingly, the yawing-moment increment of equation (12) must be modified for the elliptical lift distribution and for a variable spanwise induced upwash velocity. For this purpose yawing moments have been numerically evaluated from equation (11) for the variable induced upwash velocity corresponding to an elliptical lift distribution for model spans ranging from approximately 70 to 90 percent of the tunnel breadth, and for aileron spans from 30 to 50 percent of the model semispan, and compared with corresponding moments computed from equation (12). From these calculations the corrective coefficient K_2 has been determined as a function of the model span to tunnel-breadth ratio for several aileron spans (fig. 7) and is introduced in equation (13) to adjust the yawing-moment correction to actual loading conditions. Integrating and making the additional substitution $\Gamma_a = C_l \frac{2VSs}{a_2{}^2 - a_1{}^2}$ in equation (11), therefore gives

$$\Delta C_{n_1} = K_2 C_L \, C_l \frac{\pi S}{24b^2}\left[1 + 24 \sum_{p=1}^{\infty} \frac{p}{1 + e^{2\pi ph/b}}\right] \qquad (13)$$

where values of K_2 for any particular model are taken from figure 7. In practice it is never necessary to evaluate more than the first two terms of the infinite series.

<u>Yawing-moment correction</u> ΔC_{n_2}.-

$$\Delta C_{n_2} = \frac{1}{SV^2 s} \int_{-s}^{s} w'' \Gamma_w y dy \tag{14}$$

For a uniform lift distribution Γ_w may be replaced by $\frac{C_L VS}{4s}$ giving

$$\Delta C_{n_2} = \frac{C_L}{4Vs^2} \int_{-s}^{s} w'' y dy \tag{15}$$

Making the substitution (see equations (8) and (9))

$$\int_{-s}^{s} w'' y dy = \frac{b\Gamma_a}{4\pi} \left[F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right) \right]$$

equation (15) becomes

$$\Delta C_{n_2} = \frac{C_L b \Gamma_a}{16\pi V s^2} \left[F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right) \right] \tag{16}$$

To modify this correction to actual loading conditions a corrective coefficient (0.75), determined by evaluating numerically equation (14) for the elliptical distribution of circulation Γ_w for model spans ranging from approximately 70 to 90 percent of the tunnel breadth and comparing the results with those calculated under the assumption of constant circulation, is introduced in equation (17). Substituting for Γ_a its equivalent, equation (16) is therefore seen to become

$$\Delta C_{n_2} = 0.75\ C_L\ C_l \frac{Sb}{8\pi s(a_2^2 - a_1^2)} \left[F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right) \right] \tag{17}$$

where values of $F_1\left(\frac{a}{b}\right)$ are given in figures 5 and 6.

Yawing-moment correction ΔC_{n_3}.-

$$\Delta C_{n_3} = \frac{1}{SV^2 s} \int_{a_1}^{a_2} w'' \Gamma_a y\,dy \tag{18}$$

The indicated integration is very tedious and is therefore performed in the appendix. The numerical evaluation of the result is equally tedious for any particular model. Accordingly, numerical evaluations of the result have been made for all models of usual size in wind tunnels of the relative proportions of 7- by 10-foot and 8- by 12-foot wind tunnels and are presented graphically (figs. 8 and 9) in terms of the nondimensional parameter $F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right)$. For deflection of the right aileron alone, then, equation (18) becomes

$$\Delta C_{n_3} = -C_l^2 \frac{Ssb}{\pi(a_2^2 - a_1^2)^2} F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right) \tag{19}$$

where (cf. appendix)

$$F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right) = \frac{4\pi}{b\Gamma_a} \int_{a_1}^{a_2} w'' y\,dy$$

and values of $F_2\left(\frac{a_1}{b}\,\frac{a_2}{b}\right)$ are given in figures 8 and 9.

Total yawing-moment correction ΔC_n.- The total tunnel-wall correction to the yawing-moment coefficient is the sum of the three incremental corrections developed in the foregoing.

$$\Delta C_n = \Delta C_{n_1} + \Delta C_{n_2} + \Delta C_{n_3}$$

This correction is subtracted from the wind-tunnel-measured yawing-moment coefficient to obtain the free-air yawing-moment coefficient. Thus,

$$C_n = C_n' - \Delta C_n$$

If the NACA sign convention for airplane moments is adopted, the sign of ΔC_{n_3} is negative for up or down deflection of the right aileron alone and is positive for up or down deflection of the left aileron alone. If both ailerons are deflected equally, ΔC_{n_3} becomes zero while the corrections ΔC_{n_1} and ΔC_{n_2} remain the same.

For differential aileron deflection and positive rolling direction, ΔC_{n_1} and ΔC_{n_2} are unchanged but ΔC_{n_3} becomes

$$\Delta C_{n_3} = \left(C_{l_L}{}^2 - C_{l_R}{}^2\right) \frac{Ssb}{\pi(a_2{}^2 - a_1{}^2)^2} F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right) \qquad (20)$$

where the second subscripts refer to left and right aileron deflections, respectively. For the negative rolling direction, the sign of ΔC_{n_3} must be reversed.

ILLUSTRATIVE EXAMPLE

The following example serves to illustrate the manner in which the rolling- and yawing-moment corrections are determined for the deflection of the right aileron alone on a typical model in a 7- by 10-foot wind tunnel of closed throat. The procedure is, in general, the same for the deflection of either aileron alone or both ailerons simultaneously.

The dimensions of the typical model selected are:

Wing area, S	sq ft	10.47
Wing semispan, s	ft	3.98
Inboard aileron end location, a_1	ft	1.98
Outboard aileron end location, a_2	ft	3.81
Mean aerodynamic chord, $\bar{c}$	ft	1.45
Aspect ratio		6.1
Taper ratio		0.34

<u>Rolling-moment correction</u>.- From equation (9) the rolling-moment correction is

$$\frac{\Delta C_l}{C_l} = K_1 \frac{A}{A + 4} \frac{m_0 \bar{c} b}{4\pi(a_2{}^2 - a_1{}^2)} \left[F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right)\right]$$

For the given model dimensions,

$$K_1 = 0.69 \qquad \text{from figure 4}$$

and

$$\left.\begin{aligned} F_1\left(\frac{a_1}{b}\right) &= 0.016 \\ F_1\left(\frac{a_2}{b}\right) &= 0.142 \end{aligned}\right\} \text{from figure 5}$$

For an assumed lift-curve slope of 6.0 per radian the correction is therefore evaluated as

$$\frac{\Delta C_l}{C_l} = 0.69 \frac{6.1}{10.1} \frac{6.0\ (1.45)\ 10}{4\pi(3.81^2 - 1.98^3)} (0.142 - 0.016)$$

or

$$\frac{\Delta C_l}{C_l} = 0.034$$

From equation (10) the free-air rolling-moment coefficient is

$$C_l = \frac{C_l'}{1 + \frac{\Delta C_l}{C_l}}$$

or

$$C_l = 0.967\ C_l'$$

<u>Yawing-moment correction</u>.- The incremental yawing-moment corrections are evaluated numerically in order as follows:

(a) From equation (13)

$$\Delta C_{n_1} = K_2\ C_L\ C_l\ \frac{\pi S}{24b^2} \left[1 + 24 \sum_{p=1}^{\infty} \frac{p}{1 + e^{2\pi ph/b}}\right]$$

where, from figure 7, for the given dimensions

$$K_2 = 1.14$$

hence

$$\Delta C_{n_1} = 1.14\ C_L\ C_l\ \frac{10.47\pi}{2400}\ (1.30)$$

$$\Delta C_{n_1} = 0.020\ C_L\ C_l$$

(b) From equation (17)

$$\Delta C_{n_2} = 0.75\ C_L\ C_l\ \frac{Sb}{8\pi s\ (a_2{}^2 - a_1{}^2)} \left[F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right)\right]$$

evaluating numerically,

$$\Delta C_{n_2} = 0.75\ C_L\ C_l\ \frac{104.7\ (0.142 - 0.016)}{8\pi(3.98)(\overline{3.81}^2 - \overline{1.98}^2)}$$

$$\Delta C_{n_2} = 0.009\ C_L\ C_l$$

(c) From equation (19)

$$\Delta C_{n_3} = -C_l{}^2\ \frac{Sab}{\pi(a_2{}^2 - a_1{}^2)^2}\ F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right)$$

from figure 8

$$F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right) = 0.079$$

therefore

$$\Delta C_{n_3} = -C_l{}^2\ \frac{10.47\ (3.98)\ 10}{\pi(\overline{3.81}^2 - \overline{1.98}^2)^2}\ (0.079)$$

$$\Delta C_{n_3} = -0.094\ C_l{}^2$$

The total correction to the yawing-moment coefficient is therefore

$$\Delta C_n = 0.029\ C_L\ C_l - 0.094\ C_l^2$$

and the free-air yawing-moment coefficient is

$$C_n = C_n' - (0.029\ C_L\ C_l - 0.094\ C_l^2)$$

ACCURACY AND LIMITATIONS

The initial assumption of constant circulation over the wing has already been discussed in the theory and in the development of the corrections. The final corrections are modified for the elliptical distribution of circulation which very nearly represents the actual loading.

The additional assumption is made that the vortex sheet leaving the wing extends undisturbed an infinite distance downstream. Actually the sheet rolls up into a pair of trailing vortices in a finite distance downstream of the wing, thereby effectively shortening the span. The error introduced in the analysis of the wall interference by neglecting this effect is difficult of determination but is believed to be small in comparison with the absolute value of the corrections.

Replacing the aileron circulation by a horseshoe vortex produces an abrupt discontinuity in circulation at both ends of the aileron. Actually the increase in circulation intorduced by the deflected aileron is carried over to the wing smoothly and extends over a distance somewhat greater than the aileron span. For the normal-size model, however, this simplifying assumption is entirely justified.

When the model becomes unduly large (i.e., when its span becomes greater than 0.9 times the breadth of the tunnel) the initial assumptions probably break down seriously, affecting the accuracy of the corrections computed by this method.

CONCLUSIONS

The method of this report permits a rapid and convenient determination of the corrections to the rolling and yawing moments due to aileron deflection on models in wind tunnels with height-to-breadth ratios of 0.700 and 0.667, respectively. For wind tunnels of any other section, the general method, involving the evaluation of several complicated integral expressions, is applicable.

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APPENDIX

EVALUATION OF THE EXPRESSION

$$\int_{-s}^{s} w''y\,dy \qquad (1)$$

The velocity w'' is the net upwash velocity induced by the images of the aileron lift system; that is,

$$w'' = w''_a - w''_1$$

where w''_1 and w''_a are the velocities induced by images of the trailing vortices at $y = a_1$ and $y = a_2$, respectively. Hence (1) may be written as

$$\int_{-s}^{s} w''y\,dy = \int_{-s}^{s} w''_a y\,dy - \int_{-s}^{s} w''_1 y\,dy$$

The velocity at any point on the lifting line induced by the images of a vortex of strength Γ_a located at $y = a$

$$w''_a = \frac{\Gamma_a}{4\pi}\left\{\frac{1}{a-y} - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a-y) - \sum_{n=1}^{\infty}(-1)^n\left[\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb+(-1)^n a-y\right) - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb-(-1)^n a+y\right)\right]\right\}$$

Now let

$$F_1\left(\frac{a}{b}\right) = \frac{4\pi}{b\Gamma_a}\int_{-s}^{s} w''_a y\,dy$$

Substitution of w''_a in this expression gives

$$F_1\left(\frac{a}{b}\right) = \frac{1}{b}\int_{-s}^{s}\left\{\frac{1}{a-y} - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a-y)\right.$$

$$-\sum_{n=1}^{\infty}(-1)^n\left[\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb+(-1)^n a-y\right)\right.$$

$$\left.\left.-\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb-(-1)^n a+y\right)\right]\right\} ydy$$

or, $F_1\left(\frac{a}{b}\right) = (I_1 - I_2 - I_3 + I_4)$ (2)

where

$$I_1 = \frac{1}{b}\int_{-s}^{s}\frac{ydy}{a-y}$$

$$I_2 = \frac{1}{b}\int_{-s}^{s}\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a-y)\,ydy$$

$$I_3 = \frac{1}{b}\int_{-s}^{s}\sum_{n=1}^{\infty}(-1)^n\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb+(-1)^n a-y\right)ydy$$

$$I_4 = \frac{1}{b}\int_{-s}^{s}\sum_{n=1}^{\infty}(-1)^n\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb-(-1)^n a+y\right)ydy$$

Considering each expression in turn, it is noted first that

$$I_1 = \frac{a}{b}\ln\frac{s+a}{s-a} - \frac{2s}{b}$$

Letting $z = \frac{\pi}{h}(a-y)$

$$I_2 = \frac{h}{\pi b}\int_{\frac{\pi}{h}(a+s)}^{\frac{\pi}{h}(a-s)} z\operatorname{csch} z\,dz - \frac{a}{b}\int_{\frac{\pi}{h}(a+s)}^{\frac{\pi}{h}(a-s)}\operatorname{csch} z\,dz$$

The first integral of this expression may be evaluated by representing the function as the sum of two series and integrating the terms individually. For $z^2 < \pi^2$,

$$\int z \operatorname{csch} z \, dz = z - \frac{z^3}{18} + \frac{7z^5}{1800} - \frac{31\ z^7}{105\ 840} \cdot \cdot \cdot \qquad (3)$$

for $z^2 > \pi^2$ the function may be represented by a Taylor's series expanded about the point p such that

$$(z - p) < 1$$

Thus

$$f(z) = z \operatorname{csch} z = f(p) + f'(p)(z-p) + \frac{f''(p)}{2!}(z-p)^2 \cdot \cdot \cdot + \frac{f^n(p)}{n!}(z-p)^n$$

It may be necessary to represent the function $f(z)$ by several such series to ensure rapid convergence. For $z^2>\pi^2$

$$\int z \operatorname{csch} z \, dz = f(p)z + \frac{f'(p)}{2!}(z-p)^2 + \frac{f''(p)}{3!}(z-p)^3 \cdot \cdot \cdot + \frac{f^{n-1}(p)}{n!}(z-p)^n \qquad (4)$$

Performing the indicated integration then

$$I_2 = \frac{a}{b} \ln \frac{\tanh \frac{\pi}{2h}(s+a)}{\tanh \frac{\pi}{2h}(s-a)} + \frac{a}{b} + \frac{h}{b\pi}\left\{\frac{\left[\frac{\pi}{h}(s-a)\right]^3}{18} - \frac{7\left[\frac{\pi}{h}(s-a)\right]^5}{1800} + \frac{31\left[\frac{\pi}{h}(s-a)\right]^7}{105\ 840}\right\} - \frac{2a}{b}$$

$$+ \frac{h}{b\pi}\left[\frac{\left(\frac{\pi s}{h}\right)^3}{18} - \frac{7\left(\frac{\pi s}{h}\right)^5}{1800} + \frac{31\left(\frac{\pi s}{h}\right)^7}{105\ 840}\right]$$

$$- \frac{2h}{b\pi}\left[f(p)\frac{\pi a}{2h} + \frac{f''(p)}{3!}\left(\frac{\pi a}{2h}\right)^3 + \frac{f^4(p)}{5!}\left(\frac{\pi a}{2h}\right)^5\right]$$

I_3 and I_4 are evaluated similarly in the following:

$$I_3 = \frac{1}{b}\int_{-s}^{s} \sum_{n=1}^{\infty} (-1)^n \frac{\pi}{h} \operatorname{csch} \frac{\pi}{h} \left(nb+(-1)^n a-y\right) y\,dy$$

It can be shown that for $0.5 < \frac{h}{b} < 1.5$ terms beyond $n = 2$ are negligible and can be neglected with but a small loss in accuracy of the corrections. Therefore

$$I_3 = \left(\frac{a}{b} + 2\right) \ln \frac{\tanh \frac{\pi}{2h}(2b+a+s)}{\tanh \frac{\pi}{2h}(2b+a-s)} - \left(\frac{a}{b} - 1\right) \ln \frac{\tanh \frac{\pi}{2h}(b-a-s)}{\tanh \frac{\pi}{2h}(b-a+s)}$$

$$- \frac{2h}{b\pi}\left[\frac{\pi s}{kh}\left\{\sum_{k=1}^{k}\left[f(q_k) - f(p_k)\right]\right\} + \frac{1}{3!}\left(\frac{\pi s}{kh}\right)^3 \left\{\sum_{k=1}^{k}\left[f''(q_k) - f''(p_k)\right]\right\}\right]$$

where p is the value of z at the point about which the Taylor's series for $f(z) = z \operatorname{csch} z$ is expanded when

$$z = \frac{\pi}{h}(b-a-y)$$

and q is the corresponding point when

$$z = \frac{\pi}{h}(2b+a-y)$$

k is the number of convergent Taylor's series used to represent $f(z)$. The greater k the more rapidly the several series converge. Three series are generally sufficient.

For all practical cases it is sufficient to retain only the first four terms of the expansion (3) with no sacrifice in accuracy.

$$I_4 = \frac{1}{b}\int_{-s}^{s} \sum_{n=1}^{\infty} (-1)^n \frac{\pi}{h} \operatorname{csch} \frac{\pi}{h} \left(nb-(-1)^n a+y\right) y\,dy$$

$$I_4 = \left(\frac{a}{b} - 2\right) \ln \frac{\tanh \frac{\pi}{2h}(2b-a+s)}{\tanh \frac{\pi}{2h}(2b-a-s)} - \left(\frac{a}{b} + 1\right) \ln \frac{\tanh \frac{\pi}{2h}(b+a-s)}{\tanh \frac{\pi}{2h}(b+a+s)}$$
$$+ \frac{2h}{b\pi}\left[\frac{\pi s}{kh}\left\{\sum_{k=1}^{k}\left[f(q'_k) - f(p'_k)\right]\right\}\right.$$
$$\left. + \frac{1}{3!}\left(\frac{\pi s}{kh}\right)^3\left\{\sum_{k=1}^{k}\left[f''(q'_k) - f''(p'_k)\right]\right\}\right]$$

p' is the value of z at the point about which the series for $f(z)$ is expanded when

$$z = \frac{\pi}{h}(b+a+y)$$

and q' is the corresponding point when

$$z = \frac{\pi}{h}(2b-a+y)$$

From the foregoing development, expression (1) is seen to become

$$\int_{-s}^{s} w''y\,dy = \frac{b\Gamma_a}{4\pi}\left[F_1\left(\frac{a_2}{b}\right) - F_1\left(\frac{a_1}{b}\right)\right] \qquad (5)$$

and $F_1\left(\frac{a}{b}\right)$ may be evaluated by substituting the numerical values of the model dimensions in equation (2).

EVALUATION OF THE EXPRESSION

$$\int_{a_1}^{a_2} w''y\,dy \qquad (6)$$

Let $$F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right) = \frac{4\pi}{b\Gamma_a}\int_{a_1}^{a_2} w''y\,dy$$

Now

$$w'' = \frac{\Gamma_a}{4\pi}\left\{\frac{1}{a_2-y} - \frac{1}{a_1-y} + \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a_1-y) - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a_2-y)\right.$$

$$+ \sum_{n=1}^{\infty}(-1)^n\left[\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb + (-1)^n a_1 - y\right)\right.$$

$$- \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb + (-1)^n a_2 - y\right)$$

$$+ \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb - (-1)^n a_2 + y\right)$$

$$\left.\left. - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb - (-1)^n a_1 + y\right)\right]\right\}$$

Hence

$$F_a\left(\frac{a_1}{b}, \frac{a_2}{b}\right) = I'_1 + I'_2 + I'_3 \tag{7}$$

Where

$$I'_1 = \frac{1}{b}\int_{a_1}^{a_2}\left[\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a_1-y) - \frac{1}{a_1-y}\right] y\,dy$$

$$+ \left[\frac{1}{a_2-y} - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}(a_2-y)\right] y\,dy$$

$$I'_2 = \frac{1}{b}\int_{a_1}^{a_2}\sum_{n=1}^{\infty}(-1)^n\left[\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb+(-1)^n a_1 - y\right)\right.$$

$$\left. - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb+(-1)^n a_2 - y\right)\right] y\,dy$$

$$I'_3 = \frac{1}{b}\int_{a_1}^{a_2}\sum_{n=1}^{\infty}(-1)^n\left[\frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb-(-1)^n a_2 + y\right)\right.$$

$$\left. - \frac{\pi}{h}\operatorname{csch}\frac{\pi}{h}\left(nb-(-1)^n a_1 + y\right)\right] y\,dy$$

Evaluating the integral expressions

$$I'_1 = \frac{a_1}{b}\, ln\left[\frac{\pi\,(a_2-a_1)}{2h\ \tanh\frac{\pi(a_2-a_1)}{2h}}\right] + \frac{a_2}{b}\, ln\left[\frac{\pi\,(a_2-a_1)}{2h\ \tanh\frac{\pi(a_2-a_1)}{2h}}\right]$$

$$I'_2 = \left(\frac{a_2}{b} - 1\right)\ ln\ \frac{\tanh\frac{\pi}{2h}(b-2a_2)}{\tanh\frac{\pi}{2h}(b-a_1-a_2)}$$

$$+ \left(\frac{a_1}{b} - 1\right)\ ln\ \frac{\tanh\frac{\pi}{2h}(b-2a_1)}{\tanh\frac{\pi}{2h}(b-a_1-a_2)}$$

$$+ \left(\frac{a_1}{b} + 2\right)\ ln\ \frac{\tanh\frac{\pi}{h}b}{\tanh\frac{\pi}{2h}(2b+a_1-a_2)} + \left(\frac{a_2}{b} + 2\right)\ ln\ \frac{\tanh\frac{\pi}{h}b}{\tanh\frac{\pi}{2h}(2b-a_1+a_2)}$$

$$+ \left(2\,\frac{a_2}{b} - 1\right)\left\{\frac{\left[\frac{\pi}{h}(b-2a_2)\right]^2}{18} - \frac{7\left[\frac{\pi}{h}(b-2a_2)\right]^4}{1800} + \frac{31\left[\frac{\pi}{h}(b-2a_2)\right]^6}{105\ 840}\right\}$$

$$+ \left(2\,\frac{a_1}{b} - 1\right)\left\{\frac{\left[\frac{\pi}{h}(b-2a_1)\right]^2}{18} - \frac{7\left[\frac{\pi}{h}(b-2a_1)\right]^4}{1800} + \frac{31\left[\frac{\pi}{h}(b-2a_1)\right]^6}{105\ 840}\right\}$$

$$- 2\left(\frac{a_2+a_1}{b} - 1\right)\left\{\frac{\left[\frac{\pi}{h}(b-a_2-a_1)\right]^2}{18} - \frac{7\left[\frac{\pi}{h}(b-a_2-a_1)\right]^4}{1800} + \frac{31\left[\frac{\pi}{h}(b-a_2-a_1)\right]^6}{105\ 840}\right\}$$

$$- \frac{a_2-a_1}{b}\left\{f(m) - f(n) + \frac{\left[\frac{\pi}{2h}(a_2-a_1)\right]^2}{3!}\left[f''(m) - f''(n)\right]\right\}$$

where, as before, m is the value of z at the point about which the Taylor's series for $f(z) = z\ \mathrm{csch}\ z$ is expanded when

$$z = \frac{\pi b}{h}\left(2 - \frac{a_2-a_1}{b}\right)$$

and n is the corresponding point when

$$z = \frac{\pi b}{h}\left(2 + \frac{a_2-a_1}{b}\right)$$

That is, $m = \frac{\pi b}{h}\left(2 - \frac{a_2-a_1}{2b}\right)$ and $n = \frac{\pi b}{h}\left(2 + \frac{a_2-a_1}{2b}\right)$

Similarly,

$$I'_2 = \left(\frac{a_2}{b} + 1\right) \ln \frac{\tanh \frac{\pi}{2h}(b+2a_2)}{\tanh \frac{\pi}{2h}(b+a_1+a_2)}$$

$$+ \left(\frac{a_1}{b} + 1\right) \ln \frac{\tanh \frac{\pi}{2h}(b+2a_1)}{\tanh \frac{\pi}{2h}(b+a_1+a_2)}$$

$$+ \left(\frac{a_2}{b} - 2\right) \ln \frac{\tanh \frac{\pi}{h} b}{\tanh \frac{\pi}{2h}(2b+a_1-a_2)} + \left(\frac{a_1}{b} - 2\right) \ln \frac{\tanh \frac{\pi}{h} b}{\tanh \frac{\pi}{2h}(2b-a_1+a_2)}$$

$$+ \left(\frac{a_2-a_1}{b}\right)\left\{\left[f(u) - f(v) + f(m) - f(n)\right]\right.$$
$$\left. + \frac{\left[\frac{\pi}{2h}(a_2-a_1)\right]^2}{3!}\left[f''(u) - f''(v) + f''(m) - f''(n)\right]\right\}$$

where u is the value of z in the Taylor's series representation of $f(z) = z \operatorname{csch} z$.

$$u = \frac{\pi b}{h}\left(1 + \frac{3a_1+a_2}{2b}\right)$$

and correspondingly

$$v = \frac{\pi b}{h}\left(1 + \frac{a_1+3a_2}{2b}\right)$$

m and n are the same as defined in the evaluation of I'_2. Thus expression (6) is seen to become

$$\int_{-a_1}^{a_2} w''y\,dy = \frac{b\Gamma_a}{4\pi} F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right) \tag{8}$$

and $F_2\left(\frac{a_1}{b}, \frac{a_2}{b}\right)$ may be evaluated by substituting numerical values of the model dimensions in equation (7)

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[1]Available for reference or loan in the Office of Aeronautical Intelligence, NACA, Washington, D.C.

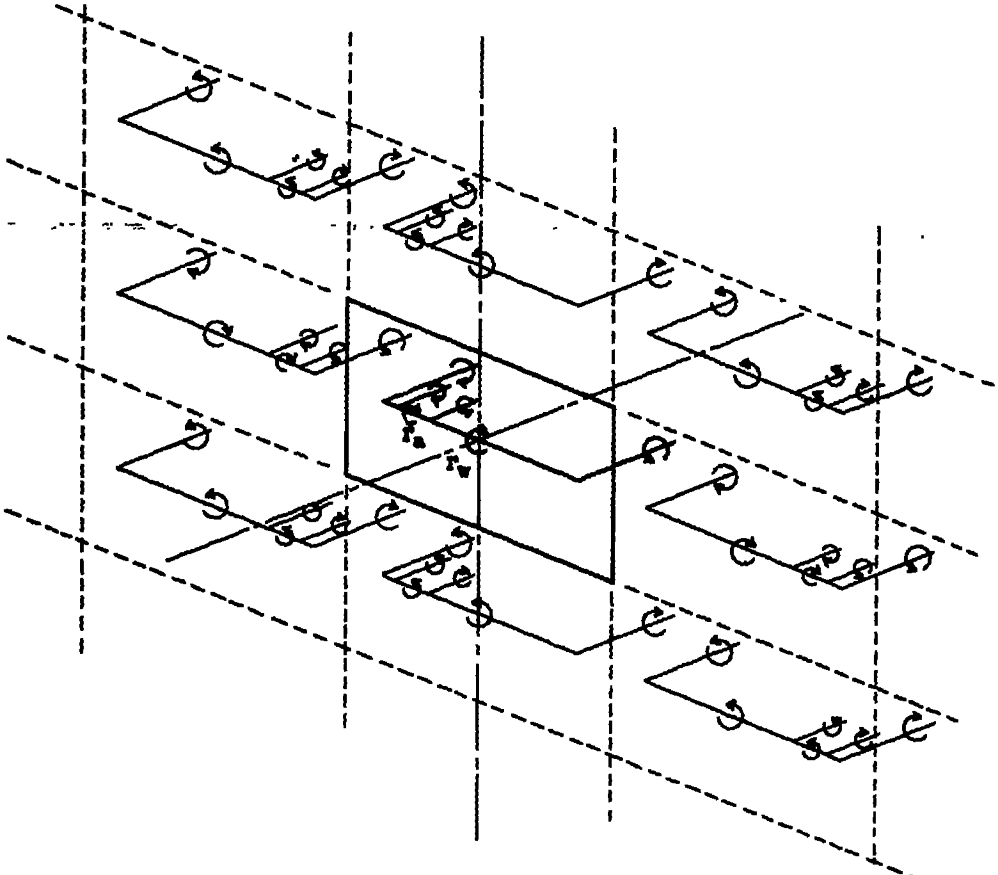
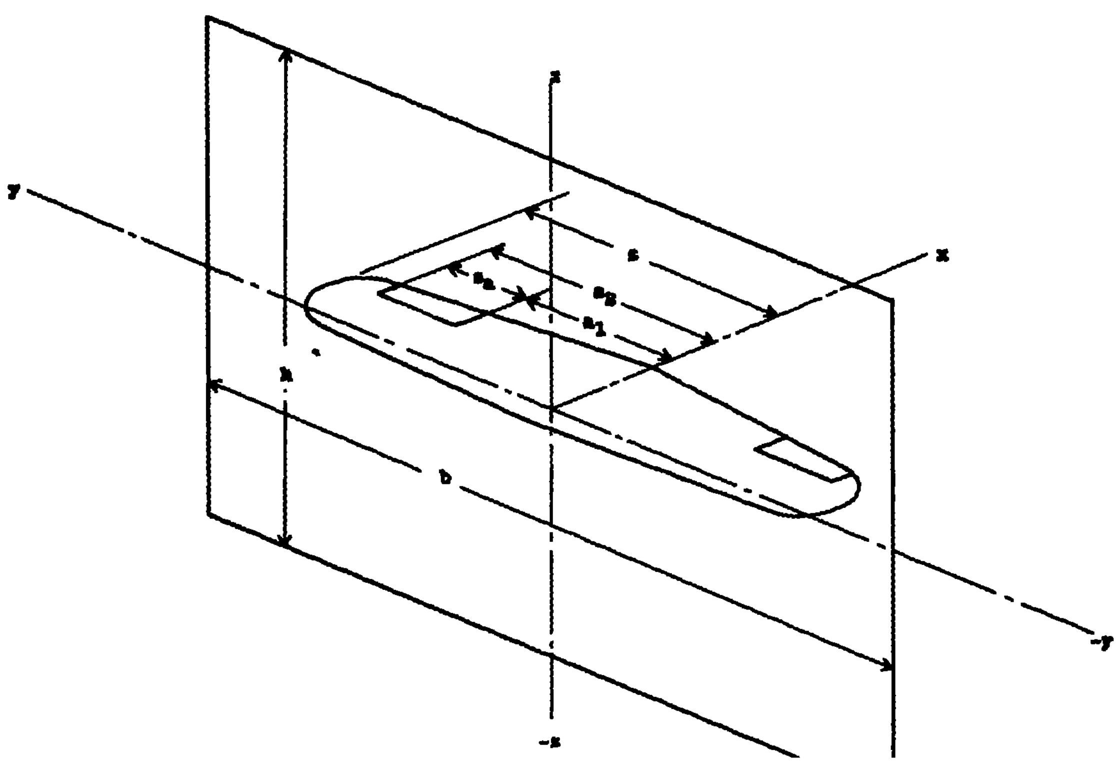


Figure 1.- Wing and aileron bound vortex system.



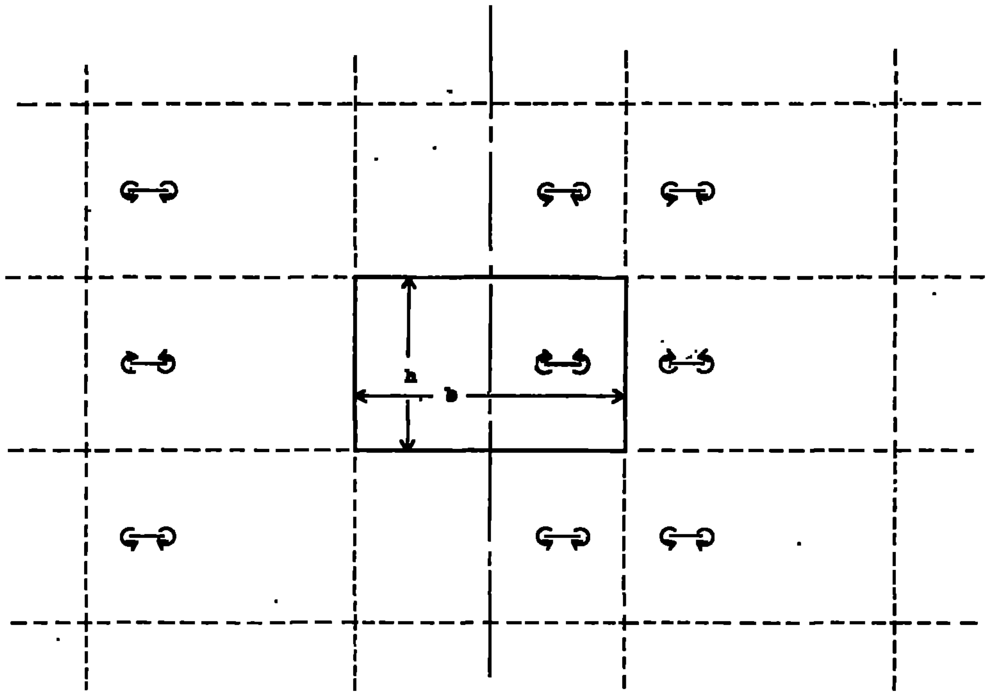


Figure 3.- Aileron image vortex system.

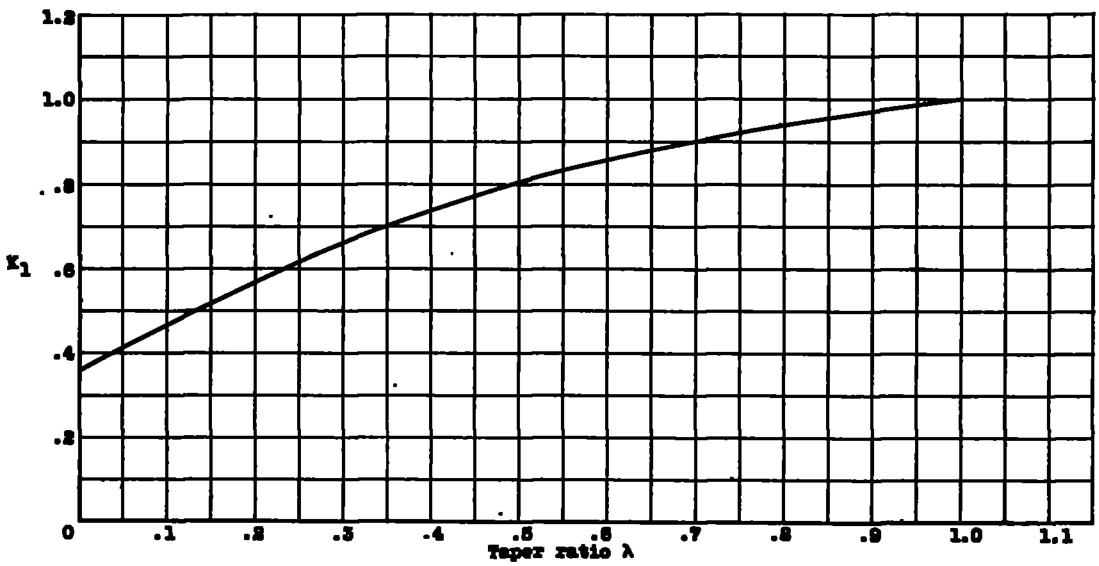


Figure 4.- Rolling-moment-correction coefficient, K_1.

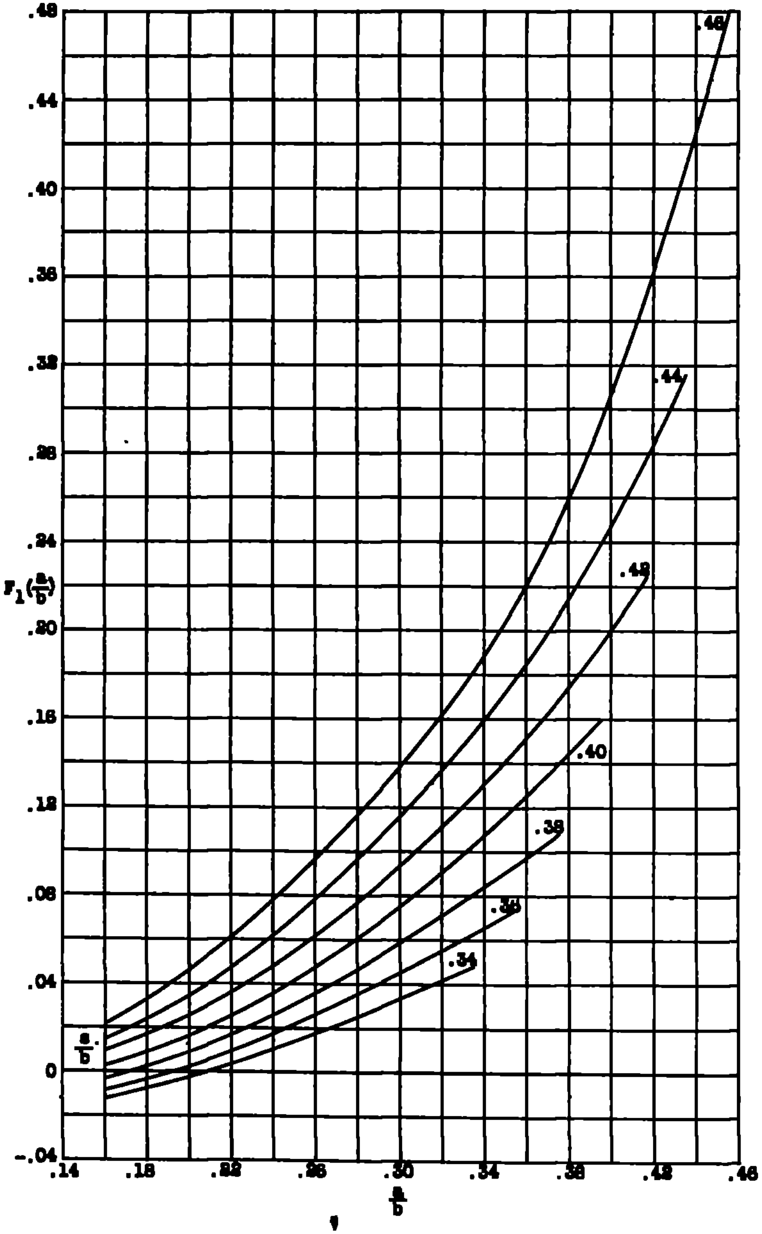


Figure 5.- Curves of $F_1(\frac{s}{b})$ for 7-by 10-foot wind tunnels.

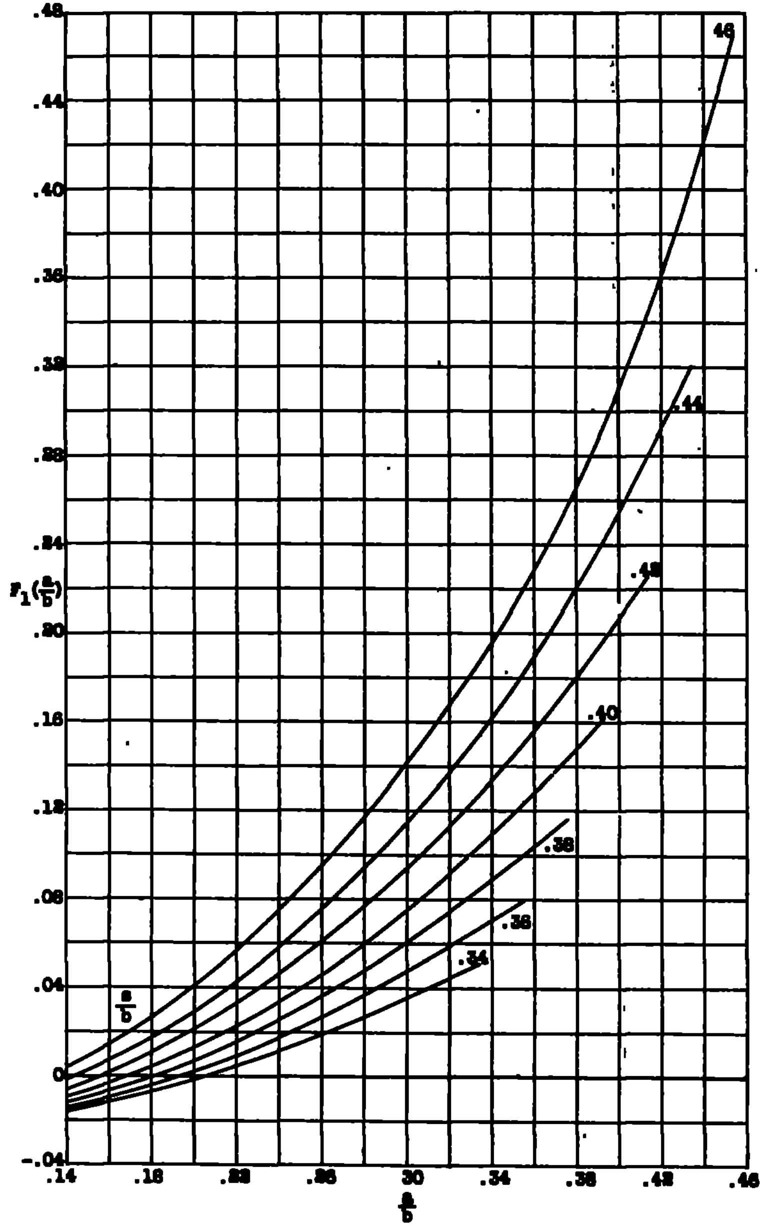


Figure 6.- Curves of $F_1(\frac{s}{b})$ for 8-by 12-foot wind tunnels.

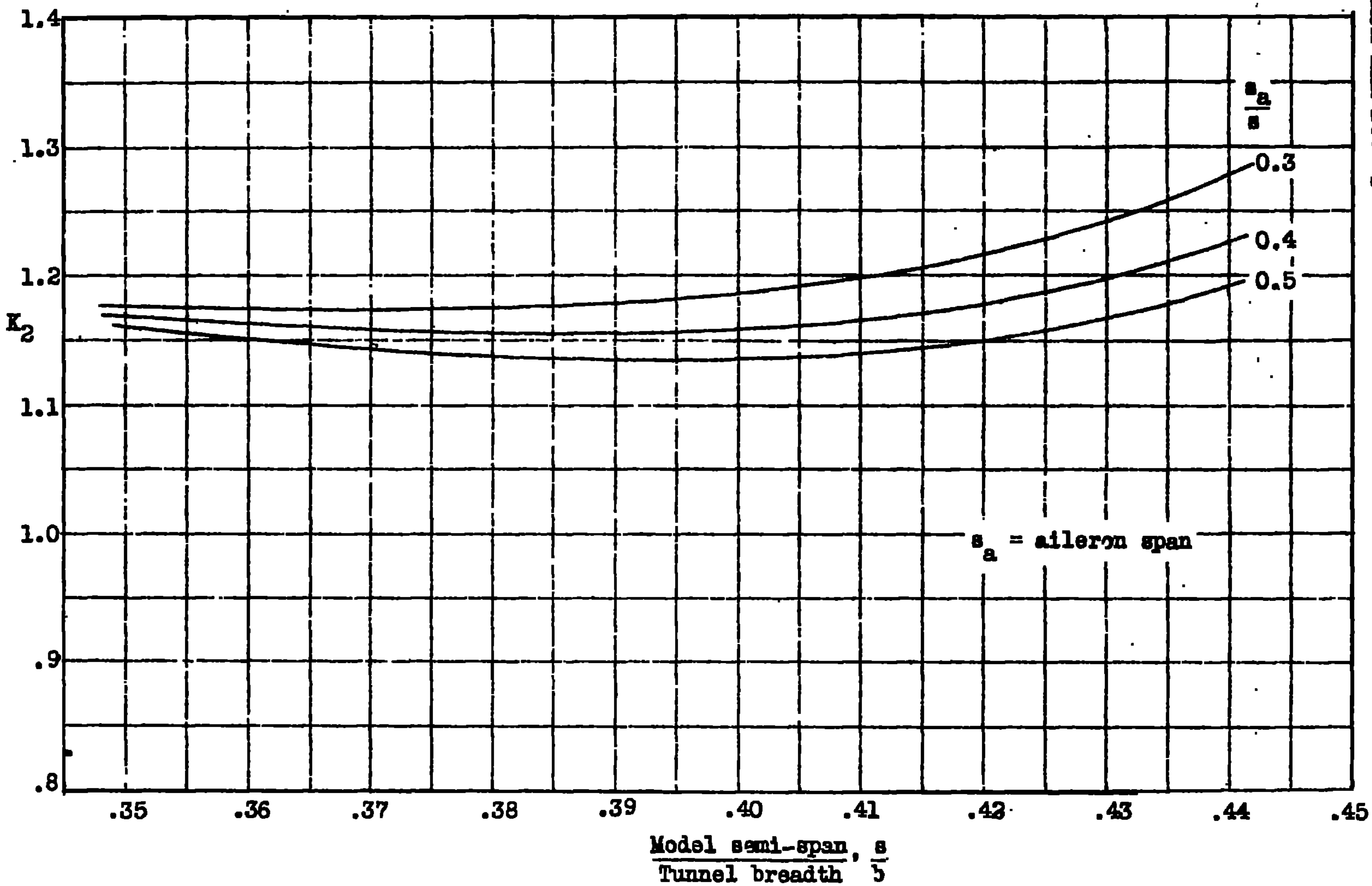


Figure 7.- Yawing-moment-correction coefficient, K_2

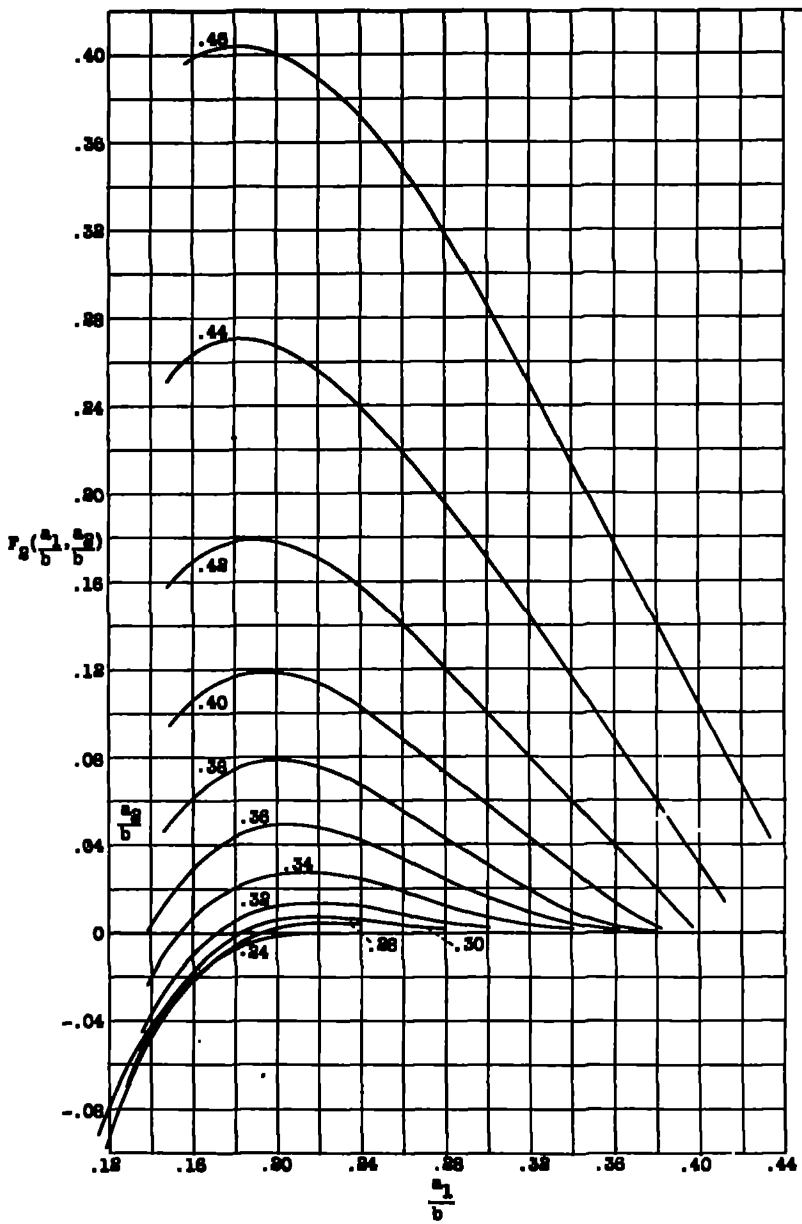


Figure 8.- Curves of $F_2(\frac{a_1}{b},\frac{a_2}{b})$ for 7-by 10-foot wind tunnels.

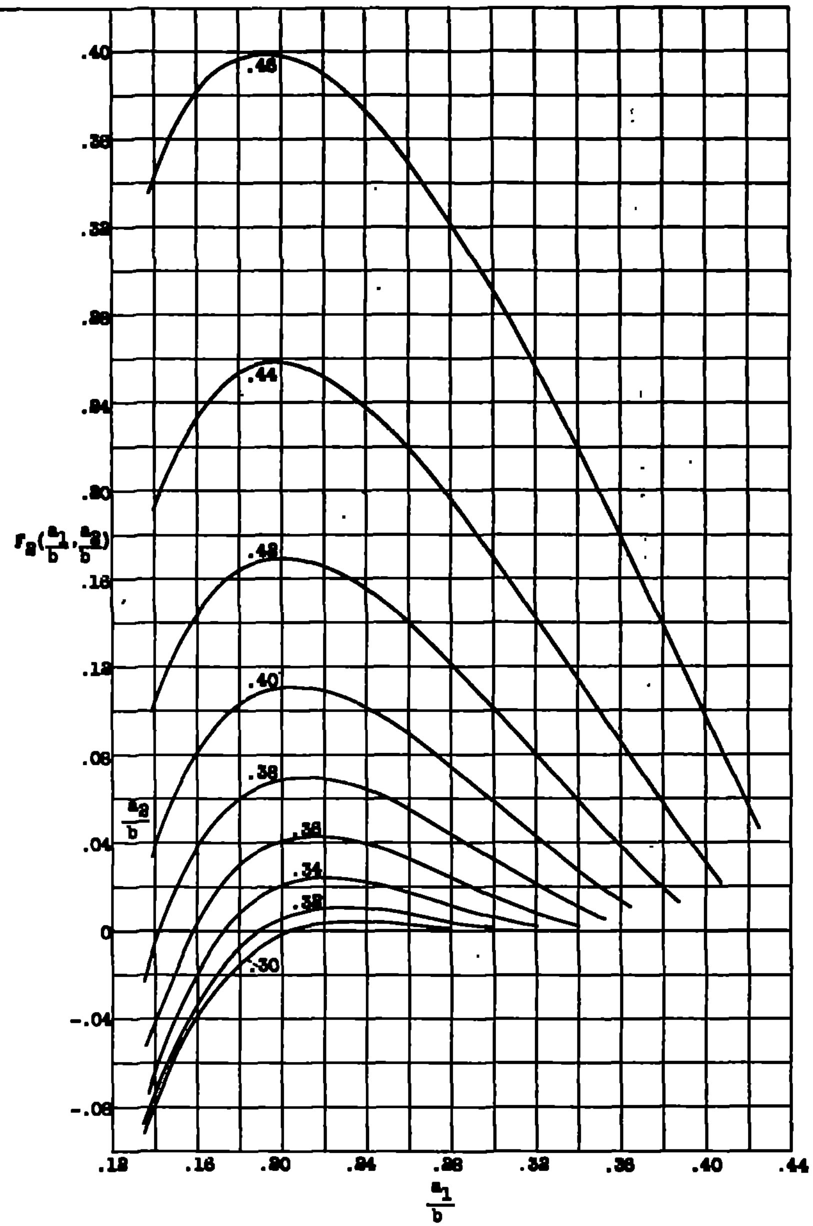


Figure 9.- Curves of $F_2(\frac{a_1}{b},\frac{a_2}{b})$ for 8-by 12-foot wind tunnels.